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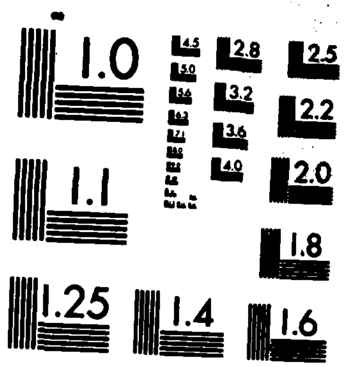

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: No. 26
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): Energy Extraction from the Electron Beam in a Free Electron Laser Resonator Gaussian Mode
5. TYPE OF REPORT & PERIOD COVERED: Technical Report 10/01/81 - 09/30/82
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): Luis R. Elias, Juan Gallardo and Peter Goldstein
8. CONTRACT OR GRANT NUMBER(s): N00014-80-C-0308
9. PERFORMING ORGANIZATION NAME AND ADDRESS: University of California, Quantum Institute, Santa Barbara, CA 93106
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61153N; RR011-07-06; NR603-001
11. CONTROLLING OFFICE NAME AND ADDRESS: ONR, 1030 E. Green St., Pasadena, CA 91106
12. REPORT DATE: 1983
13. NUMBER OF PAGES:
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): Unclassified/Unlimited
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution.

DTIC ELECTE FEB 2 0 1986 S D

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES: Proceedings of the International Conference of Lasers '82, 1983.
19. KEY WORDS (Continue on reverse side if necessary and identify by block number): Extraction, Beam, Gaussian
20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

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SANTA BARBARA

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FREE ELECTRON LASER PROJECT

Energy Extraction from the Electron Beam
in a Free Electron Laser Resonator Gaussian Mode

Luis Elias, Juan Gallardo, and Peter Goldstein

Lasers '83

QIFEL026/83

ENERGY EXTRACTION FROM THE ELECTRON BEAM IN A FREE ELECTRON LASER RESONATOR GAUSSIAN MODE

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ABSTRACT

We present preliminary analytical results of energy extraction and gain of a Free Electron Laser (FEL) in the presence of a TEMoo Gaussian mode. Our result is applicable to all values of the parameter q (length of the undulator L divided by the Rayleigh range z_R) in second order of the input field. We also present simple analytical approximate expressions for the small signal gain valid for $q \leq 2.8$.

Introduction

Free Electron Lasers are based on the process of wave amplifications by high quality relativistic electron beams.[1] The electron beam passes through a periodic electromagnetic structure (undulator) and amplifies a co-propagating wave. The most common input wave is a TEMoo gaussian mode generated by either an external laser in the amplifier configuration or excited within the oscillator. A schematic diagram of a Free Electron Laser amplifier is shown in Fig. 1.

Most previous work on Free Electron Lasers assumes plane wave radiation fields. This approach does not account for variations in phase and amplitude of the electromagnetic wave with z. It also requires an ad-hoc filling factor due to the finite size of both the optical and electron beam.

A more realistic approach using Gaussian electromagnetic waves incorporates the varying phase and amplitude. This leads to a semi-analytic filling factor and to a gain curve different from that obtained in the plane wave analysis.

The electron beam is assumed to be filamentary with an initially uniform charge distribution in the longitudinal direction. The position of each electron within an optical wavelength is described the the random phase ϕ_o. The combined effect of a finite cross section electron beam and Gaussian TEMoo mode is to introduce a filling factor $FF \sim q\ell_{elec}/L$ in the gain expression.

Three-dimensional numerical treatments of the FEL problem has been published by several groups.[2] They show that the radiation field consists mainly of a Gaussian TEMoo mode modified slightly by higher order radiation modes. For this reason this paper deals only with the amplification of a TEMoo mode.

In this paper we derive an analytic expression for the gain in second order of the input electric field E_o, with no restrictions on the parameter $q(=L/z_R)$. We also present an approximate expression which is compact, simple and accurate for $q \lesssim 2.8$. We note the possibile application to optimize cavity design.

Electron Dynamics and Energy Loss

We assume a circularly polarized wiggler magnetic field:

$$\vec{B} = B_o\,(\cos k_oz,\ \sin k_oz,\ 0) \qquad \text{with} \qquad k_o = 2\pi/\lambda_o$$

Our gaussian TEMoo input field is

$$\vec{E} = E_o\frac{\exp[-r^2/\omega(z)^2]}{\omega(z)/\omega_o}\left(\cos(kz-\omega t+\psi),\ \sin(kz-\omega t+\psi),\ 0\right)$$

where $\omega(z) = \omega_o\sqrt{1+(z/z_R)^2}$ is the beam radius, and ω_o is the beam waist at the center of the resonator (z=0). The phase is $\psi = -\tan^{-1}(z/z_R)$ with radius of curvature $R(z) = z(1+(z_R/z)^2)$ and Rayleigh range $z_R = \pi\omega_o^2/\lambda$. To a very good approximation the transverse velocity is given by

$$\vec{v} = -\frac{eB_o}{k_o\gamma_o mc^2}\left(\cos(k_oz+\phi_o),\ \sin(k_oz+\phi_o),\ 0\right)$$

where $\gamma_o mc^2$ is the initial electron energy, ε_o, and ϕ_o is the initial phase of the electrons.

From the Lorentz equations of motion

$$\frac{d\gamma}{dz} = -\frac{e}{mc}\,\vec{v}\cdot\vec{E} = \frac{eKE_o}{\gamma_o mc}\,\mathrm{Re}\left\{\frac{\exp[i(k+k_o)z - i\omega t + i\phi_o]}{1+i\left(\frac{z}{z_R}\right)}\right\}$$

where we have used

$$\frac{\exp[-i\tan^{-1}(z/z_R)]}{\sqrt{1+\left(\frac{z}{z_R}\right)^2}} = \frac{1}{1+i\left(\frac{z}{z_R}\right)}$$

with

$$K = \frac{eB_o}{k_o mc^2}$$

The electrons follow a nearly helical trajectory so we let $z = c\beta_o t + \delta z$ where $\beta_o = \beta_{res} + \delta\beta$ is the initial electron velocity, we also define $\eta = (k+k_o)c\delta\beta t$, so that the phase can be written as

$$(k+k_o)z - \omega t + \psi_o = \eta + (k+k_o)\delta z + \psi_o$$

We then integrate our equation of motion with the definitions $\psi(t=-L/2c) = \psi_o$, $\eta_o = (k+k_o)\delta\beta\, z_R/\beta_{res}$ and $\eta_i = (k+k_o)\delta\beta L/2$, which yields

$$\Delta\gamma = \gamma(t) - \gamma_o = \frac{eKE_o z_R}{\gamma_o mc^2 \eta_o \beta_{res}} \mathrm{Re}\left\{\int_{-\eta_i}^{\eta_i} d\eta' \, \frac{\exp[i(\eta'+(k+k_o)\delta z+\psi_o)]}{1 + i\, z/z_R}\right\} \quad (1)$$

For low optical power density it is appropriate to expand all dynamical variables in a power series in E_o.

$$\gamma = \gamma_o + \sum_{n=1} \Delta\gamma^{(n)}$$

$$z = c\beta_o t + \delta z = c\beta_o t + \sum_1 \delta z^{(n)}$$

where $\beta_o ct$ is independent of E_o and δz contains higher powers of E_o. The lowest order change in the electron energy is

$$\Delta\gamma^{(1)} = \frac{eKE_o z_R}{\gamma_o mc^2 \eta_o \beta_{res}} \mathrm{Re}\left\{\int_{-\eta_i}^{\eta_i} d\eta' \, \frac{\exp[i(\eta'+\psi_o)]}{1+i\eta'/\eta_o}\right\} \quad (2)$$

where we have used

$$\frac{c\beta_o t}{z_R} = \frac{\eta}{\eta_o}\left(1 + \frac{\delta\beta}{\beta_{res}}\right) \approx \frac{\eta}{\eta_o} \quad .$$

If we take the real part of the integral in Eq. 2 and let $x = \eta/\eta_o$; $x_i = \eta_i/\eta_o$ we find

$$\Delta\gamma^{(1)} = \frac{eKE_o z_R}{\gamma_o mc^2 \beta_{res}} \cos\psi_o \int_{-x_i}^{x} dx' \, \frac{\cos\eta_o x' + x'\sin\eta_o x'}{1+x'^2} \quad (3)$$

To compute the average energy loss per particle of an initially uniform beam we average over the phase ψ_o. As is expected $\langle\Delta\gamma^{(1)}\rangle_{\psi_o} = 0$. To compute $\langle\Delta\gamma^{(2)}\rangle_{\psi_o}$ we use the Madey theorem $\langle\gamma_F - \gamma_i\rangle = \frac{1}{2}\frac{d}{d\gamma_i}\langle(\gamma_F - \gamma_i)^2\rangle$. Where $\gamma_i = \gamma_o$ is the initial electron energy, and γ_F is the energy at the end of the resonator.

Equating like powers of E_o

$$\langle\Delta\gamma^{(2)}\rangle_{\psi_o} = \frac{1}{2}\frac{d}{d\gamma_i}\langle(\Delta\gamma^{(1)})^2\rangle_{\psi_o}$$

If we define [illegible] $= 2\eta_i$, $\xi = \frac{L}{z_R}$, and [illegible] $= \xi q$ where [illegible] is the resonance parameter, we find

$$\langle \Delta\gamma^{(2)} \rangle = -\frac{e^4 B_o^2 E_o^2 L^3}{k_o (\varepsilon_o)^3 mc^2} \frac{I_1 I_2}{q^3} \tag{4}$$

with

$$I_1 = \int_{-q/2}^{q/2} dx \; \frac{\cos \alpha x + x \sin \alpha x}{1 + x^2} \; ; \qquad I_2 = \int_{-q/2}^{q/2} dx \; \frac{x \sin \alpha x - x^2 \cos \alpha x}{1 + x^2}$$

These integrals are easy to compute numerically and we have done so using a composite Simpson method. Alternatively we can rewrite these integrals in terms of the imaginary part of the complex exponential integral[4] $E_1(x)$.

$$I_1 = 2e^{-\alpha} \operatorname{Im}\left\{ E_1\left(-\alpha + i\alpha\frac{q}{2}\right)\right\}$$

$$I_2 = 2 \frac{\sin \alpha q/2}{\alpha} + I_1$$

Thus

$$\langle \Delta\gamma^{(2)} \rangle = \frac{e^4 B_o^2 E_o^2 L^3}{k_o \varepsilon_o^3 mc^2} \left(\frac{4}{q^3} \frac{d}{d\alpha} \left[e^{-\alpha} \operatorname{Im}\left\{ E_1\left(-\alpha + i\alpha \frac{q}{2}\right)\right\}\right]^2 \right) \tag{5}$$

We define the gain as minus the average energy loss of the electron beam; divided by the signal field energy.

$$G = -\frac{\int dV \, \rho_e \langle \Delta\gamma^{(2)} \rangle \, mc^2}{\int dV \, U}$$

where U is the signal field energy density and ρ_e is the electron number density. With Σ_e equal to the cross sectional area of the electron beam we have for a constant density beam

$$G = e^4 B_o^2 \lambda_o \left(\frac{L}{\varepsilon_o}\right)^3 \frac{4I}{ec\lambda L} \left\{ \frac{4}{q^2} \frac{d}{d\alpha} \left[e^{-\alpha} \operatorname{Im} E_1\left(-\alpha + i\alpha\frac{q}{2}\right)\right]^2 \right\} \tag{6}$$

where I is the current of the electron beam. In Fig. 2 we have plotted the curly bracket for different values of q which we call the unnormalized gain. We note that the gain curve shifts to higher values of the resonance parameter and the absorption peak becomes smaller than the gain peak as q increases.

Figures 3 and 4 show ν_{max} and $G_{max}(\nu_{max}, q)$ as a function of q. Both show an approximately linear behavior with q for $0 \lesssim q \lesssim 2.8$, i.e., $\nu_{max} \approx 2.6 + q$. In the limit of small q we have

$$I_1 = \left(1 - \frac{d}{d\alpha}\right) \frac{2 \sin \alpha\frac{q}{2}}{\alpha} \approx q \, \frac{\sin \frac{\nu - q}{2}}{\frac{\nu - q}{2}}$$

and

$$\frac{I_1 I_2}{q^3} = -\frac{1}{2} \frac{d}{d\nu} \left(\frac{\sin \frac{\nu - q}{2}}{\frac{\nu - q}{2}} \right)^2$$

so that

$$G = e^4 B_o^2 \lambda_o \rho_e \left(\frac{L}{\varepsilon_o}\right)^3 \left(4 \frac{\Sigma_e}{\lambda L} q\right) \left\{ \frac{1}{2} \frac{d}{d\nu} \left(\frac{\sin \frac{\nu - q}{2}}{\frac{\nu - q}{2}} \right)^2 \right\} \tag{7}$$

which is the plane wave result[3] modified by the filling factor $4 \frac{\Sigma_e q}{\lambda L}$ and the $\frac{\nu - q}{2}$ dependence of the curly bracket term.

The results displayed in Figs. 2, 3, and 4 allow us to expand the region of validity of Eq. (7) originally derived for $q \approx 0$, to a larger range of values; $0 \le q \le 2.8$.

When $\nu = 0$ the integrals in G are easy to compute

$$I_1 = 2\tan^{-1}\frac{q}{2} \quad ; \quad I_2 = 2\tan^{-1}\frac{q}{2} - q$$

thus

$$G(0,q) = e^4 B_0^2 \rho_e \lambda_0 \left(\frac{L}{\epsilon_0}\right)^3 \left(4\,\frac{\sum_e}{\lambda L}\,q\right) \frac{4}{q^3}\ \tan^{-1}\frac{q}{2}\left(\tan^{-1}\frac{q}{2} - \frac{q}{2}\right) \qquad (8)$$

As shown in Fig. 4 at $q \sim 4$, the amplitude of the gain curve is maximized; this is also verified in Fig. 5 where we plot G(0,q) the gain at zero resonance parameter. We see that as q increases the gain becomes more negative reaching a minimum at $q \simeq 4$ and then increasing slowly for higher values of q.

Discussion and Conclusions

Our results are useful in the design of FEL cavities. The optimal design corresponds to $q \approx 4$ which gives maximum gain. This value of q can be obtained by appropriately choosing the mirror's radius of curvature R(z) (i.e., z_R) and the length of the resonator and the undulator L.

Other suggested procedures for optimizing resonator design are: a) minimizing the optical energy within the undulator, which yields $q = 2\sqrt{3}$; b) insisting that there be good overlap of the radiation field and input gaussian beam[7] i.e., $\theta_{rad} \sim (0.75)/\gamma\sqrt{N} = \lambda/\pi w_0$. This gives $q \simeq 4.7$ for the UCSB experiment.

In conclusion, for a filamentary beam we have derived an analytic small signal gain expression for arbitrary q (see Eq. 6). A plot of this function is presented in Fig. 2. We also provide a gain formula in the limit of small q, shown in Eq. 7. This is the 1-d small-signal gain expression shifted to higher values of the resonance parameter by an amount q; furthermore, we suggest that this well-known compact and simple formula can be extended to a wider range $0 \le q \le 2.8$.

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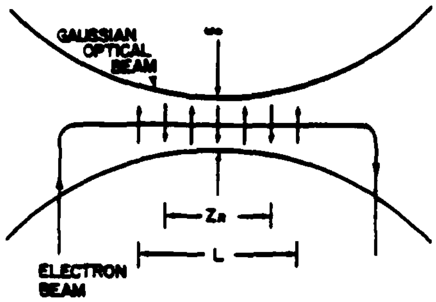

Figure 1. Schematic of a FEL amplifier.

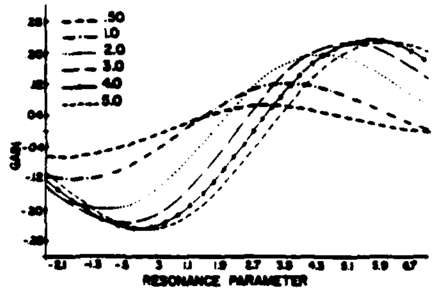

Figure 2. Unnormalized small signal gain (curly bracket in Eq. 6) vs. resonance parameter for several values of the parameter $q=L/z_R$.

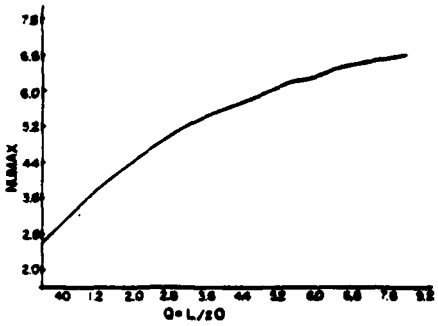

Figure 3. $_{max}$ value of the resonance parameter corresponding to the maximum of the gain curve as a function of q.

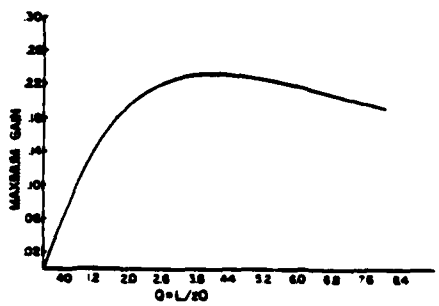

Figure 4. Maximum of the small-signal gain for different values of parameter q.

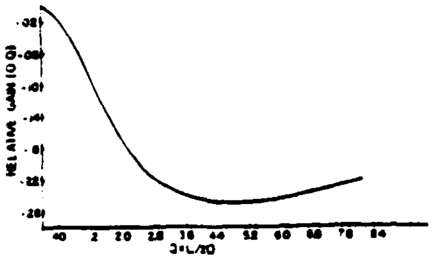

Figure 5. Plot of Eq. 8 showing the unnormalized gain at =0 as a function of q.

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